ChatGPT-4 & Questions from a Materials Thermodynamics Course

Question 1: Fundamental Equations & Equations of State

TLL Question 1:

Find 3 equations of state for a system with the fundamental equation (K is a constant):

U = K S^3 N^-1 V^-1

ChatGPT4 answer:

The fundamental equation you've provided describes the internal energy U of a system as a function of entropy S, number of particles N, and volume V, with K as a constant. We can use this equation to derive the equations of state by calculating the partial derivatives of U with respect to S, N, and V, holding the other variables constant.

The equations of state of a system are commonly given by the temperature T, pressure P, and chemical potential μ, which are related to the internal energy U as follows:

Temperature T is the rate of change of internal energy with respect to entropy at constant volume and number of particles:
T = $(\partial U/\partial S)$_N,V

Pressure P is the negative rate of change of internal energy with respect to volume at constant entropy and number of particles:
P = -$(\partial U/\partial V)$_S,N

Chemical potential μ is the rate of change of internal energy with respect to the number of particles at constant entropy and volume:
μ = $(\partial U/\partial N)$_S,V

With U = K S^3 N^-1 V^-1, let's find these derivatives:

Temperature:

T = $(\partial U/\partial S)$_N,V = $\partial/\partial S$ [K S^3 N^-1 V^-1] = 3 K S^2 N^-1 V^-1

Pressure:

P = -$(\partial U/\partial V)$_S,N = - $\partial/\partial V$ [K S^3 N^-1 V^-1] = K S^3 N^-1 V^-2

Chemical Potential:

μ = $(\partial U/\partial N)$_S,V = $\partial/\partial N$ [K S^3 N^-1 V^-1] = - K S^3 N^-2 V^-1

So, the equations of state for your system are:

T = 3 K S^2 N^-1 V^-1
P = K S^3 N^-1 V^-2
μ = - K S^3 N^-2 V^-1

TLL follow-up:

Are these equations of state more practically useful than the initial fundamental equation ?

ChatGPT4 answer:

The fundamental equation of a thermodynamic system encapsulates all the thermodynamic information about that system. However, in practice, it's often more useful to work with the equations of state.

The reason is that the equations of state express the relationships between the thermodynamic variables that we can more easily control or measure. Here are some reasons why these equations might be more practical:

Temperature (T): It's easy to set the temperature of a system or measure it directly, and many physical and chemical properties change in predictable ways with temperature.

Pressure (P): It's also straightforward to set or measure the pressure, and like temperature, many properties vary in predictable ways with pressure.

Chemical Potential (μ): This is more complex to measure directly but is crucial in systems where the number of particles can change, like in chemical reactions or phase transitions.

Thus, the equations of state are more practical because they tell us how to adjust the temperature, pressure, or chemical potential to achieve the desired changes in the system's state. In comparison, the fundamental equation provides a more abstract representation of the system's thermodynamics, without clearly stating how to control or measure the variables involved.

Nonetheless, the fundamental equation is crucial because it's the starting point for deriving these more practical equations of state. Without it, we wouldn't have a complete understanding of the thermodynamics of the system.

TLL follow-up:

How can I use the T equation of state if I don't have a direct measurement of entropy ?

ChatGPT4 answer:

The relationship $T = 3 K S^2 N^{-1} V^{-1}$ gives you an explicit formula for temperature as a function of entropy, number of particles, and volume, with K as a constant. However, as you pointed out, in practical applications, it can be challenging to measure entropy directly.

While you can't measure entropy directly, in many cases you can infer its changes. For example, you often know how much heat was transferred to a system at a known temperature, and from that, you can calculate the change in entropy since $\Delta S = Q/T$ for a reversible process.

Additionally, the entropy is often included in the state variables because it is a fundamental quantity in thermodynamics and statistical mechanics. In those contexts, the S in the equation could be thought of as the "unknown" that you're solving for.

However, in many practical applications, you would likely use tables or models to get the entropy values. Many thermodynamics textbooks and resources provide extensive tables for the entropies of various substances at different temperatures and pressures.

Finally, in some cases, it might be possible to re-arrange or re-express your equations of state to get rid of the explicit dependence on S, replacing it with variables that are easier to measure. However, this would depend on the specifics of the system and the processes involved.

In any case, while this specific equation of state might seem less practical due to the direct dependence on entropy, remember that it is one piece of a larger thermodynamic puzzle. It is still valuable in a theoretical sense because it helps describe how temperature changes with entropy, given constant number of particles and volume.